CS 6100 Homework 4
10 Points

Note, these exercises may be done in groups of one, two, or three. If more than one person is involved, list all the names on ONE set of answers. Groups may change throughout the term. Working in groups is a BIG plus for you. Take advantage of it. If you work in groups, you must work in the group for the ENTIRE assignment. It is considered cheating if you work with someone else for some of the answers, but turn in an individual copy of the answers. It is an all or nothing situation. You can't work together on some questions and alone on some. Sometimes I see an individual whose name is listed in two groups. This is strictly forbidden and is considered cheating. You cannot work in two groups.

1. Discuss how the revelation principle relates the Vickrey auction to the English auction.
2. Show an example where an agent is best off bidding insincerely if a Vickrey auction is implemented as "open cry" instead of sealed bid.
3. Multi-unit auctions. There are m identical items to be auctioned off to n players. Each player is single-minded (wants only the bundle specified) and wants q_i items at a total value v_i (more than q_i items give the same value). Since the bidders are single-minded, the bids are of the form (p_i, b_i) where p_i is an integer in 1..m that indicates the number of items of interest and b_i is the bid for them (assume b_i are rational numbers).

Example: There are 50 identical items. The bids are as follows:

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Number wanted</th>
<th>Bid for bundle</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>11</td>
<td>19</td>
</tr>
<tr>
<td>B</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>24</td>
<td>58</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>E</td>
<td>18</td>
<td>36</td>
</tr>
<tr>
<td>F</td>
<td>27</td>
<td>62</td>
</tr>
<tr>
<td>G</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td>H</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>

a. Is this problem a difficult one to solve in terms of complexity?

b. Develop a simple greedy type algorithm to solve the problem

4. Second price auction with entrance fee

There are n bidders in the auction. The valuations of a single buyer are independent and uniformly distributed on the interval [0; 1]. (See the notes in Chapter 14 for a discussion of what would be done in a first price auction without entrance fees.) To participate in a second price auction, every bidder has to pay an entrance fee of e > 0.

a) How will a participating bidder (who already paid the entrance fee) bid?

b) Which bidders participate in the auction and pay the entrance fee? Hint: Assume that there exists a valuation v such that bidders with a higher valuation than v participate in the auction whereas bidders with lower valuation do not. A bidder with precisely valuation v is indifferent between participation and non participation.
c) In your opinion, which entrance fee maximizes the seller's profit? Should the seller introduce an optimal reservation price or an optimal entrance price?

5. Prove that truth telling is the dominant strategy for Vickrey auctions.

6. Why is the Vickrey auction design, which is so "lovely in theory, so lonely in practice"?

7. Consider the following example of values for multiple items. Using Vickrey Clarke Groves, who wins what items and what do they pay?

Agent 1: ({a}, 3), ({b}, 3), ({a, b}, 10)
Agent 2: ({a}, 3), ({b}, 0), ({c}, 5)
Agent 3: ({a}, 0), ({b}, 2), ({c}, 3)
Agent 4: ({a}, 0), ({a,b}, 7), ({a, b, c},13)

8. Dan is auctioning an object that Alice, Bob and Charlie want to acquire. They all know that the value of the object to each of the three is uniformly and independently distributed between 0 and 100. That is, the value of the object to Alice is $Va$, to Bob is $Vb$ and to Charlie is $Vc$, where $Va$, $Vb$, $Vc$ are independent random variables uniformly distributed between 0 and 100. Each person knows its own value. Alice gets a payoff of 0 if she does not acquire the object and a payoff of $Va - p$ if she does, where $p$ is the price she pays. Payoffs to Charlie and Bob are similarly defined. Assume that all players are risk-neutral.

(a) Suppose Dan conducts a first price sealed bid auction. Further, suppose all three players, Alice, Bob, and Charlie chooses the same strategy such that the strategies are in equilibrium. What price should Alice write on her sealed bid? (Note that this price can depend on $Va$, which Alice knows, but not on $Vb$ or $Vc$.)

(b) What price should she bid if Dan conducts a second price sealed bid auction?

(c) For this part, suppose $Va = 80$. If Dan conducts a Dutch auction, at what price should Alice jump in and stop the auction?

(d) Under each of the above auction types, how much should Dan expect to obtain for the object?