CS 6890 Homework 3 (20 points)

Written homework provides an excellent framework for achieving the goals of this course. Because assignments are done as a group and any questions are discussed in class or during office hours, written solutions to the homework will not be provided. These are typed exercises, but you are certainly encouraged to actually program some of them. Be sure to show your work for all the problems. Note, these exercises may be done in groups of one or two (or with instructor approval, three). If more than one person is involved, list all the names on ONE set of answers. Groups may change throughout the semester. Answers should not be compared with others not in your group.

1. Consider the bimatrix game below: Find the (interior) Nash equilibrium with mixed strategies. Do the problem two ways. Once with partial derivatives (like on page 71) and once the way we first used in class [by setting each players pure strategy utilities equal to each other].

   \[ A = \begin{pmatrix} 0 & 3 \\ 2 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}. \]

2. Consider the bimatrix game below: Find the (interior) Nash equilibrium with mixed strategies. Do the problem two ways. Once with partial derivatives (like on page 71) and once the way we first used in class. You haven’t seen an example quite like this, but its just more of the 2x2 case. The book has some examples.

   \[ A = \begin{pmatrix} 0 & 2 \\ 1 & 0 \\ 3 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 0 \\ 2 & 1 \\ 0 & 1 \end{pmatrix}. \]

3. Find the Nash equilibrium of a two-person strategic form game with strategy sets \( S_1 = S_2 = \mathbb{R} \)

   \[ u_1(x, y) = y^2 - x^2 - 2x + y \quad \text{and} \quad u_2(x, y) = 2x^2 - xy - 3y^2 - 3x + 4y. \]

4. Apply iterated elimination of dominated strategies to the following three person game for players \( r, c, \) and \( t. \) When a strategy is dominated, indicated ALL strategies that dominate it and whether the domination is strict or weak. Recall, weak domination means that all entries are greater or equal to all choices of the dominated option. Make sure you eliminate all dominated strategies before reducing the game. Indicate whether or not you can solve the game using this method. If it can be solved, list the order of elimination of strategies. Assume the payoffs represent \( (r, c, t). \)

\[
\begin{array}{c|cc|c|cc|c}
& c_1 & c_2 & & c_1 & c_2 \\
\hline
r_1 & 0, 0, 1 & 5, 3, 4 & & 3, 4, 3 & 1, -2, 5 \\
r_2 & 2, 1, 0 & 1, 2, -2 & & 4, -1, 2 & 0, -2, -1 \\
\end{array}
\]
5. For each of the zero sum games below, use the saddle point idea of page 49 to determine all optimal strategies. If no pure strategy exists, use a mixed strategy.

\[
\begin{array}{ccc}
-1 & 0 & 2 \\
3 & 1 & 1 \\
0 & 1 & 2 \\
\end{array}
\quad
\begin{array}{ccc}
-4 & 0 & 3 & 4 & 2 \\
-6 & 1 & -2 & 3 & 2 \\
-1 & 4 & 0 & 0 & 2 \\
\end{array}
\quad
\begin{array}{ccc}
2 & 4 & 2 \\
1 & 2 & 2 \\
2 & 3 & 2 \\
\end{array}
\]

6. Find an optimal strategy for the zero sum game below:

\[
\begin{array}{c}
2 \\
-2 \\
\end{array}
\quad
\begin{array}{c}
-1 \\
3 \\
\end{array}
\]

7. Sophie now has $100 to spend. Movies are $3 each and games are $4 each. If she rents M movies and G games, her utility is $M^2 + G^2$.

a. Draw the indifference curves and the budget line (as in the notes). *I posted a spread sheet which may help with the values.*

b. Solve the problem using LaGrange and verify that your solution is correct.

c. If the indifference curves are concave, does the optimal solution still lie tangent to the budget line (as it did for convex indifference curves)?

8. Based on the texts problem #1, page 93. For families with two children, assume that each point in the sample space is equally likely. $S=$\{BB, BG, GB, GG\}.

a. What is the probability of two girls?

b. Given that one child is a girl, what is the conditional probability that the other is a girl?

9. An event subset B $\subseteq$ S with $p(B) > 0$ is fixed. Calculate $p(A|B)$ if

(a) $A \cap B = \emptyset$

(b) $B \subseteq A$

(c) $p(A,B) = p(A)p(B)$  (Note: In this case, A and B are called independent events.)

10. (Page 93 Problem 5) Estimates show that 0.3% (less than 1/3 percent) of the U.S. population is carrying HIV. In order to study the spread of HIV, it was suggested that the U.S. Congress pass a law requiring that couples applying for a marriage license should take a blood test for HIV. The blood test is considered very “effective” since

a. A person with HIV has a 95% chance to test positive.

b. A person without HIV has a 4% chance to test positive.

After several lengthy discussions, it was determined that the HIV test was ineffective for determining the spread of AIDS disease and its implementation was abandoned. What is the probability a person will test positive (ie., $p(B)$). What is the probability you will test positive if you have the disease? Can you figure out what argument persuaded the legislators to abandon the plans for the test?