Yale Lectures 21 and 22

21. Repeated Games: Cooperation vs the End Game

Most relationships are not contractual. Between nations, no court to take it to.

- In ongoing relationships, the promise of future rewards and the threat of future punishments MAY sometimes provide incentives for good behavior today. But for this to work, it helps to have a future.
- End effects: lame duck, retirement, economics major’s relationships

- Repeated interaction does not induce cooperation in a finite period game.

- We can sustain equilibrium even if finite game, but it requires multiple equilibrium, can use one as a reward and one as a punishment. Can use that difference to encourage cooperation today.
• If a stage game has more than one NE, we may be able to use the prospects of playing different equilibria tomorrow to provide incentives today.

• If we play repeated games, we can discount future earnings by delta like we saw in the shrinking pie problems (delta is the probability that there will be a future).

• In addition, we look at the gain today from defecting against the probability of future penalty.

**Lecture 22 - Repeated games: cheating, punishment, and outsourcing**

How to achieve cooperation without contracts?

Need: gain if cheat today <= value of relationship after cooperation (promise) – value of relationship after cheating (threat)

Need threat to be believable.
Prisoner’s Dilemma

Known ends cause problems – as we won’t keep threat or promise, as we will just play the NE.

Credible threats and credible promises – we look at subgame perfect equilibria as they hold in every subgame.

What doesn’t work. Coop forever – as no reason to stick to it.

When is grim trigger an equilibrium? If the temptation today is less than value of promise forever – value of threat forever.

<table>
<thead>
<tr>
<th>Temptation today</th>
<th>C</th>
<th>D</th>
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<td>3-2 &lt; [value of promise – value of threat][tomorrow (weight by delta)]</td>
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Value of Promise (C,C forever)

Value of threat (D,D forever)

3-2 < value of promise – 0

\[
3-2 < 2 + 2\delta + 2\delta^2 + 2\delta^3 + \ldots
\]

Geometric progression.

\[
x = \frac{2}{1 - \delta}
\]

\[
1 < \frac{2}{1 - \delta} \ast \delta \text{ as punishment may not happen}
\]

so is in equilibrium if \( \delta \geq 1/3 \)

This is only one way of defecting, but if other person is playing D, no reason for me to do otherwise.
Lessons:

Lessons: Grim strategy works because the play it works when cooperate and the play it suggests when defect are BOTH NE. We need the probability of continuation to be high enough. We can get cooperation in PD with grim trigger as a SPE provided \( \delta \) is big enough.

In general: For an ongoing relationship to provide incentives for good behavior today, it helps for there to be a high probability that the relationship will continue. The probability that it will continue is the weight you put on the future. The more weight I put on the future, the more probability I cooperate today.

Grim trigger has some disadvantages in the real world

One Period Punishment

Let's try another one. One period punishment. If either cheats, both punish for next period, and then both cooperate. If both cooperate or both defect, cooperate next.

If we repeat analysis, it is stable if \( \delta \geq \frac{1}{2} \). So shorter punishments means we need more weight on the future.