**Lectures 7&8**

Revisit Elections
- Now range is left-wing or right-wing politics. Candidate-Voter model
- Assume even distribution of voters. Voters will vote for closest candidate.
- Change some critical assumptions of model.
- Number of candidates is not fixed – adjusts itself
- Candidates cannot select their positions.
- Assume each voter is a potential candidate.
- Strategy – run or not to run
- Voters vote for closest running candidate

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**Payoff**

- Payoff – prize for winning (b)
- cost of running (c)
- \( b \geq 2c \)
- unhappiness if candidate who wins is far from me \(-|x-y|\) (you are at x, winner is at y)
- Payoff for x – if x wins, b-c
  - if x loses but y wins, -c - |x-y|
  - if x doesn’t run, - |x-y|

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**Where are NE?**

Demonstrate guess and check method of finding NE.

- No one runs – not NE
- Single person runs in center – NE
- If two center candidates, not NE (win with prob 50%). One could drop out. One on other side would win as didn’t have to split votes.
- If two center candidates not identical, a third candidate could lose and cause winner to be further away.
- Candidates at 1/6+e, 5/6-e (center candidate cannot win)

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**Lessons to learn**

- Many NE, not all at center
- A third voter could lose and cause winner to be further away
- If the two candidates are in at the extremes, someone at the center will enter and win
- Guess and check method for finding an equilibrium works pretty well.

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**Where to live**

- Two types of people
- Two Towns – East and West
- Pick where to live simultaneously
- If there’s no room in a town, then we allocate the surplus randomly.
- Do best with mixed town, but if unbalanced, better to be in majority.

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**Payoff Function**

What are equilibria?
- a. If exactly split, no reason to change. Worrisome as small chance of getting there. If we deviate, we don’t get hurt much. Indifferent. It isn’t strongly reinforced. At complete segregation, strongly reinforced. If we move away from equilibrium, falls apart.
- Weak equilibrium, not east to get to. Population would prefer being in this equilibrium. Likely to end up where less preferred.
- b. Two segregated equilibria in E and S in W, or vice versa. If completely segregated, no reason to change.
- c. What if all pick E – would be divided randomly. This seemingly irrelevant detail of game, created a new NE. If randomization was available, we are going to be very close to integration. We would be better off. Having society randomise for you is better than active choice.
Lessons

1. Segregation resulted when it isn’t preferred. Seeing segregation does not imply there is a preference for segregation.
2. Policy (randomization – a bit like busing)
3. Each person could randomly choose and arrive at equilibrium

Mixed Strategy

• When would you mix?
• What strategies would be involved in the mix?
• How would you compute the mix?

Rock Paper Scissors – MIXED strategy

• No pure strategy equilibria
• Get stable state by randomizing
• See payoffs
• Expected payoff (Rock against \(1/3, 1/3, 1/3\)) – will get 0.
• Expected payoff (Paper against \(1/3, 1/3, 1/3\)) – will get 0.
• Expected payoff (Scissors against \(1/3, 1/3, 1/3\)) – will get 0.
• Expected payoff (1/3, 1/3, 1/3) against \(1/3, 1/3, 1/3\) (a third of the time I am playing rock against \(1/3, 1/3, 1/3\) which we know the expected payoff is zero).
• So opponent has no reason to change to improve his score, BUT if he doesn’t randomize, the person picking \(1/3, 1/3, 1/3\) will not stay in that choice.