CS7910 Homework 7

1. Does the revenue equivalence theorem hold even when bidders are risk-averse? Explain.
2. Is the Vickrey Clarke Groves mechanism vulnerable to collusion? Explain.
3. A combinatorial auction for edges in a graph. Suppose we are selling edges in an undirected graph (perhaps representing network capacity). Each bidder i is interested in obtaining a set of edges that constitute a path from some source node si to some target node ti. Any path will give the bidder the same value vi. Receiving more than one path from si to ti is worthless (i.e. it will still give the bidder a total value of only vi). Thus, a bid takes the form (si; ti; vi).
   For example, consider the following graph.
   ![Graph Image]
   Suppose we receive the following bids:
   Bidder 1: (A;E; 4)
   Bidder 2: (C; F; 2)
   Bidder 3: (B;E; 1)
   Then, the optimal allocation is to give edges AB and BE to bidder 1, and edges CD and DF to bidder 2, for a total value of 4 + 2 = 6. Note that it is impossible to accept bidder 3's bid in addition: there are only two paths from B to E, namely the one consisting of edge BE, and the one consisting of edges BC, CD, and DE. Since edges BE and CD have been allocated already, neither of these paths can be allocated to bidder 3.
   a. Compute the VCG payments for bidders 1 and 2.
   b. Express each bidder's bid using the XOR language, i.e. as an XOR over bundles of edges (with values for each bundle).
   c. In general graphs, is it possible that you will need an exponential number of XORs to express one of these source-target valuations? Justify your answer with an example. Exponential means exponential in the number of nodes.

4. Bayesian games – given different probabilities for each game being played. Player 1’s type selects between the row games. Player 2’s type selects between the column games.

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(a) Show the induced normal form if the players do not know their type or their opponent’s type.

(b) Show the induced normal form if player 1 knows its type. Note, this will be two tables. One if he knows he is type 1 and a second if he knows he is type 2. [I reposted the notes on this, as they were flawed.]

5. Auctions without a seller. Consider the following valuations of four bidders for the same item.

\[
\begin{align*}
V_1(A) &= 10 \\
V_2(A) &= 3 \\
V_3(A) &= 5 \\
V_4(A) &= 9
\end{align*}
\]

Who wins and what does each pay/get using Bailey, Porter, and Cavallo?

6. Consider incremental elicitation. What multistage mechanisms have we seen already?

7. Coalitions. Suppose we have the following values of coalitions.

\[
\begin{align*}
v\{1\} &= 3 \\
v\{2\} &= 1 \\
v\{3\} &= 2 \\
v\{1,2\} &= 5 \\
v\{2,3\} &= 5 \\
v\{1,3\} &= 7 \\
v\{1,2,3\} &= 10
\end{align*}
\]

How should we divide the profit if they all work together?

- List a solution which is feasible, fair, efficient, stable?
- List a solution which is not feasible.
- List a solution which is not fair.
- List a solution which is not efficient.
- List a solution which is not stable.

8. Coalitions. Suppose we have the following values of coalitions.

\[
\begin{align*}
v\{1\} &= 3 \\
v\{2\} &= 2 \\
v\{3\} &= 2 \\
v\{1,2\} &= 4 \\
v\{2,3\} &= 7 \\
v\{1,3\} &= 7 \\
v\{1,2,3\} &= 10
\end{align*}
\]

What would the Shapley solution give to each player? Is it in the core? Explain.


Consider a combinatorial auction with free disposal that uses the VCG mechanism. In this auction, all but one of the bidders are well-behaved. The false-name bidder can use an unlimited number of false names (bid as several different players), and knows the bids of the well-behaved bidders. In what cases, can the false-name bidder obtain all items for free?