Surface-embedded Passive RF Exteroception: Kepler, Greed, and Buffon's Needle

Vladimir Kulyukin, Aliasgar Kutiyanawala, and Minghui Jiang

Computer Science Assistive Technology Laboratory (CSATL)
Department of Computer Science
Utah State University
Logan, UT, 84322
`vladimir.kulyukin@usu.edu, {aliasgar,mjiang}@cc.usu.edu`

Abstract. Surface-embedded passive radio frequency (PRF) exteroception is a method whereby an action to be executed by a mobile unit is selected through a signal received from a surface-embedded external passive RFID transponder. This paper describes how Kepler's hexagonal packing pattern is used to embed passive RFID transponders into a carpet to create PRF surfaces. Proof-of-concepts experiments are presented that show how such surfaces enable mobile robots to reliably accomplish point-to-point navigation indoors and outdoors. Two greedy algorithms are presented for automated design of PRF surfaces. A theoretical extension of the classic Buffon's Needle problem from computational geometry is presented as a possible way to optimize the packing of RF transponders on a surface.

1 Introduction

A smart environment is a regular everyday environment, e.g. a home, a store, or a community center, instrumented with embedded sensors and computer systems that that make use of the data they receive from those sensors in order to support a quality-of-life function. The University of Washington Assisted Cognition Project [17] seeks to synthesize AI and ubiquitous computing to develop solutions that help people with cognitive limitations. Japan's Ministry of Land, Infrastructure, and Transport announced its support for the Autonomous Movement Support Project [18] whose objective is to embed small electronic sensors into the pavement and street furniture to supply users with location-specific information anytime and anywhere. Willis and Helal [15] propose an assisted navigation system where an RFID reader is embedded into a blind navigator's shoe and passive RFID sensors are placed in the floor.

Mobile units that operate in smart environments utilize either *proprioception* (action is determined relative to an internal frame of reference) or *exteroception* (action is determined from a stimulus originating in the environment itself). Low power requirements, low cost, and ease of installation are among the principal reasons for a wide acceptance of RFID as an exteroceptive technology in many application domains [20]. Kantor and Singh use RFID tags for robot localization and mapping[6]. Once the positions of the RFID tags are known, their system uses time-of-arrival type of information to estimate the distance from detected tags. Tsukiyama[7] developed a navigation system for mobile robots using RFID tags under the assumption of perfect signal reception and zero uncertainty. Hahnel et al.[10] developed a probabilistic robotic mapping and localization system to analyze whether RFID can be used to improve the localization of mobile robots in office environments.

Since smart environments are composed of surfaces [16], it is natural to pose the question of how PRF sensors can be embedded into those surfaces in order to improve the point-to-point navigation and localization of mobile units operating in those environments. This paper describes how Kepler's hexagonal packing pattern is used to embed passive RFID transponders into horizontal surfaces. Proof-of-concept experiments show how such surfaces enable mobile robots to accomplish point-to-point navigation indoors and outdoors. Two greedy algorithms are presented for automated design of PRF surfaces. Simulations show that greed compares favorably to brute force and hill climbing. An extension of the classic Buffon's Needle problem from computational geometry is proposed as a possible way to optimize the packing of PRF transponders on a surface. An optimal two-column pattern of arranging transponders on the edges of a surface is briefly investigated.

2 The Problem

A RFID reader reads a RFID transponder, through it antenna, by powering it with electromagnetic waves. A factor that determines whether a tag can be read or not is the number of electromagnetic lines of force that pass through the coil of the antenna. This is a function of the proximity of the antenna to the tag and its orientation with respect to the tag.

Tag collision is another factor. If two tags are near an antenna, both may be powered up and transmit their identification codes simultaneously. Most readers do not have a collision avoidance mechanism and as a result no tags are read. Thus, the read area of an antenna is also a function of the proximity of the tags with respect to each other.

The above considerations lead to the following formulation of the problem. Given a mobile device capable of carrying a number of RFID antennas, what type of PRF surface would be needed for the device to accomplish point-to-point navigation tasks reliably. Furhermore, to what extent does surface-embedded PRF exteroception simplify on-board computational machinery and increase navigational reliability?

The above considerations lead to the following formulation of the problem. Given a mobile unit capable of carrying a number of RFID antennas, what type of PRF surface is needed for the unit to accomplish point-to-point navigation reliably. The problem can be motivated through two application scenarios. The first application is robot-assisted navigation for the blind and cognitively impaired [14, 11, 12]. Safety is the primary requirement. The robot's pose must be known with absolute certainty to determine the next navigation action. It is not feasible to assume, as is generally assumed in many probabilistic approaches [5, 10], that there can be a period of time when the robot is not certain about its pose, but can recover from the uncertainty by navigating the environment. To be sure, the environment must be instrumented with PRF surfaces, which incurs a cost. However, the overall cost is reduced, because the on-board navigation machinery is likely to become simpler.

The second application is rapidly deployable transportation infrastructures for autonomous vehicles. In an urban disaster area, it is critical to have an infrastructure for evacuating the sick and wounded. Probabilistic approaches may be inappropriate because of the high cost of calibration: there simply may not be sufficient time to move the robot around all intended routes and let it build an adequate sensor map. In addition, other exteroceptive sensors, such as GPS, are highly suspect in urban canyons and in areas for which no GPS maps exist. On the other hand, a network of PRF surfaces can be rapidly deployed, manually or through a teleoperated vehicle, in order to establish a temporary transporation infrastructure which, after the mission is completed, can be removed and deployed elsewhere. A critical point is that the cost of calibration is equivalent to the cost of deployment, because no subsequent fine tuning is needed.

3 A Solution

3.1 Where can a tag be read?

As was mentioned above, the area where a tag can be read by the antenna, called *read area*, depends on the distance between the tag and the antenna, the orientation of the antenna with respect to the tag, and the proximity of other tags. To get a better understanding of a tag's read area, experiments were performed for four different orientations of the antenna with respect to a single tag at a fixed vertical distance of 3cm from the tag.

The RFID tag used in the experiments is the wedge shaped transponder (RI-TRP-W9WK) from Texas Instruments, Inc. The reader used in the experiments is the Series 2000 reader (RI-STU-MB2A) from Texas Instruments, Inc. It operates on 134.2 kHz. This reader was also chosen due to its small size, ease of operation and compatibility with the selected tag type. The antenna used is the Stick Antenna (RI-ANT-PO2A) from Texas Instruments, Inc.

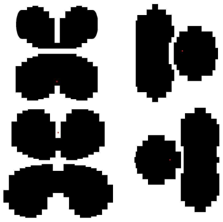
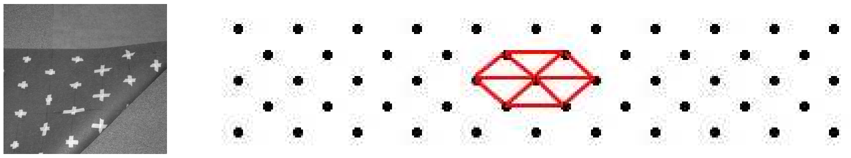


Fig. 1. Read Shapes at Four Orientations.

Figure 1 shows the areas (in black) where the tag (designated by circles) can be read and areas (in white) where the tag cannot be read by the antenna raised 3cm from the tag at the following orientations: 0 degrees (upper left), 90 degrees (upper right), 180 degrees (bottom left), and 270 degrees (bottom right). The black shapes, although irregular, can be approximated with regular shapes, e.g., circles or squares, which, as discussed below, bodes well for the automated design of PRF surfaces.

3.2 PRF Suface

In his book *De Nive Sexangula* (On the Six-Sided Snowflake), Kepler asserted that in the 3D space, face-centered cubic packing, e.g., apples on a fruit stand, was the tightest possible. Approximately 200 years later, Axel Thue proved the conjecture for the 2D space. Thue's theorem states that no packing of overlapping discs of equal size in the plane has denisity higher than that of the hexagonal packing [19]. Since the read area of a tag can be approximated as a disc, Thue's theorem immediately applies.



(a) Carpet (b) Pattern

Fig. 2. (a) PRF Carpet (b) Hexagonal Packing Pattern.

A total of 280 tags were placed beneath a standard carpet surface, 4 meters long and 2 meters wide, in a hexagonal pattern in which each tag is 15cm from its neighbors shown in Figure 2(b). Figure 2(a) shows a small section of the carpet surface with embedded tags. The carpet forms its own 2D coordinate system where each tag is mapped to its x,y coordinates. The distance of 15cm was discovered experimentally to be the smallest distance that does not result in overlapping between the tag read areas. As the density of tag packing increases, up to a point, the localization resolution increases but so does the cost. When the packing of tags becomes too dense, many ID collisions must be resolved and the localization resolution decreases.

4 Proof-of-Concept Experiments

Figure 3(a) shows the platform used in the first experiment. This robot has a differential drive mechanism that allows it to move forward, backward as well as turn in place. It is equipped with two RFID readers and antennas and a microcontroller for interfacing them with an on-board laptop. The robot was placed on a PRF surface (2 meters by 4 meters). The navigation task was to patrol the surfaces's perimeter. The robot did a total of five 10 minute patrols without going off the mat or deviating off the planned paths that consisted sequences of tag IDs.

(a) Tiger Robot (b) Pioneer 2DX

Fig. 3. (a) Tiger Robot Indoors (b) Pioneer 2DX Outdoors.

In the second experiment, the Pioneer 2DX robotic base from Activmedia Robotics, Inc. was used. It was also equipped with two RFID readers and antennas. The point of this experiment was to demonstrate the rapid deployability of PRF surfaces. A PRF surface (0.75 meters by 2.5 meters) on a sidewalk on the Main Quad of Utah State University and had the robot patrol the surface. The robot did a total of five 10-minute patrols without going off the surface or deviating from the planned paths.

5 Automated Design of PRF Surfaces

It is desirable to automate the design of PRF surfaces to reduce cost and improve localization. Before describing the algorithms for automating PRF suface design, several assumptions must be explicitly stated.

5.1 Assumptions

The read area area of a tag is assumed to be a circle with a known radius centered on the tag. It is also assumed that collision resolution is not available. Thus, if the readable areas of two tags intersect, neither tag can be read in that area.

A mobile device operating on an PRF surface is assumed to have a fixed number of RFID readers and placed according to a fixed pattern. Let R be the number of RFID readers placed on the unit. The inclusion of each RFID reader should increase the probability of localization as well as the robustness of the entire system. Let n^a denote the number of available tags and n denote the actual number of tags used. Localization probability as well as robustness of the system should increase with an increase in the number of tags.

Figure 4 shows the surface into which available tags must be packed. It is discretized into N points (shown by blue colored dots) on both sides of the surface. It is assumed that the mobile device can cross the surface only along a straight line (shown by blue colored lines) connecting any two points on either side of the surface. The device is said to be moving along a *valid path* when it travels along a straight line between any two of the N points on either side of the surface.

The probability of localization, P, is defined to be the probability of a RFID reader intersecting the readable area of a tag, when the device is moving along a valid path.

Let T be the total number of valid paths on the surface. Then $T = N^2$. Let S be the number of paths that intersect with the readable area of at least one tag. Then $P = \frac{S}{T}$.

5.2 Algorithms

Automated design of PRF surfaces can now be formualted as an optimization problem: Position a given number of circles on a surface of a given area in such a way that the circules do not intersect with each other but intersect with the maximum number of valid lines on the surface. By maximizing the number of lines that intersect the circles, the probability of localization is maximized. Four different algorithms were developed and implemented to solve this problem.

– **Brute-Force Algorithm:** All feasible packing patterns to position the circles on the surface are enumerated and the probability of localization is computed for each pattern by dividing the number of lines that intersect with the circles with the total number of lines on the surface. The pattern that maximizes the localization probability is chosen. Ties are broken arbitrarily. The algorithm is exponential in the number of tags. Only simluations for surfaces with 2 and 3 tags completed. Simulations for four or more tags were not observed to terminate after several days of compuation. Even though this algorithm is not practical, it can serve as a baseline to compare the results of the other algorithms for the cases of two and three tags.
– **Static Greed:** The maximum number of valid lines intersect the circles, if the circles are placed at line intersections. For example, if only one circle was available, it would be placed in the center of the surface to maximize the localization probability. Initially, all unique points where the valid lines intersect are calculated and weighed according to the number of lines that pass through them. Figure 5.2 shows the unique intersection points (represented by red colored dots) along with their weights for a surface with $N = 3$ points and $T = 9$ lines. The intersection points are sorted in descending order of their weights. The circles are placed on the highest available intersection point until the available number of tags is exhaused or it is no longer possible to place any more circles on the surface.
– **Dynamic Greed:** The static greedy algorithm chooses the weights of the points (which are computed only once at the start of the algorithm) as the basis for placing the next circle and fails to consider that a line may already be covered by a previously placed circle. This problem is rectified through dynamic recomputation of the intersection weights each time a circle is placed. After each placement, the lines that are already covered with circles are taken out.
– **Hill-Climbing Method:** All available circles are thrown randomly on the surface in such a way that their readable areas do not intersect. The probability of localization is computed. A circle is chosen at random and moved in a random direction by a random distance in such a way that the readable areas do not intersect. If the value increases, the move is accpeted, otherwise it is rejected.

5.3 Simulations

Simulation experiments were performed to compare the above algorithms. A simulated surface with $N = 15$ points was used. The results of the experiments are summarized

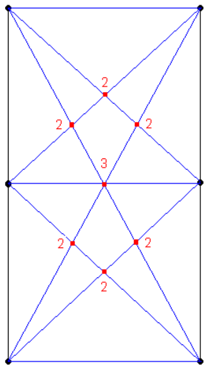


Fig. 4. RFID mat with valid paths and intersection points

in Figures 5(a) through 7(b). The blue colored circle represents the area where a tag can be read, green colored lines represent localized paths and red colored lines represent unlocalized paths. The ratio of the number of green colored lines to the total number of lines gives the probability of localization.

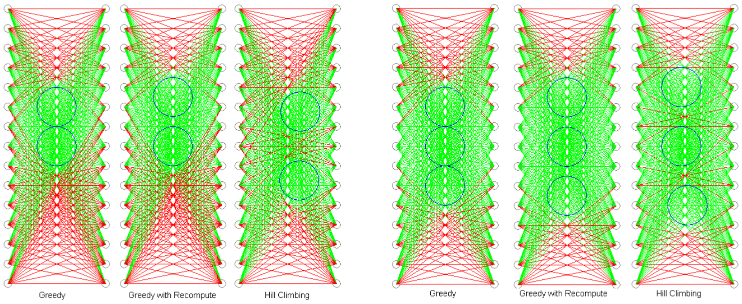
The experiments showed that dynamic greed is the best solution of the four. Even though the brute-force method gives the provably best results, it is impractical to use. Hill-Climbing is better for relatively smaller number of tags but the results of the dynamic greed algorithm are better in real-life situations where surfaces are more densely packed.

Table 1. Probability of localization (in %) for different algorithms and number of tags

Number of Tags	Brute-Force	Greedy	Greedy with Recompute	Hill Climbing (average)
2	63.11	53.77	53.78	58.56
3	78.66	69.77	76.89	73.32
4	-	80.44	85.78	84.32
5	-	91.11	94.67	90.56
6	-	95.55	97.33	94.93
7	-	95.55	100	97.58

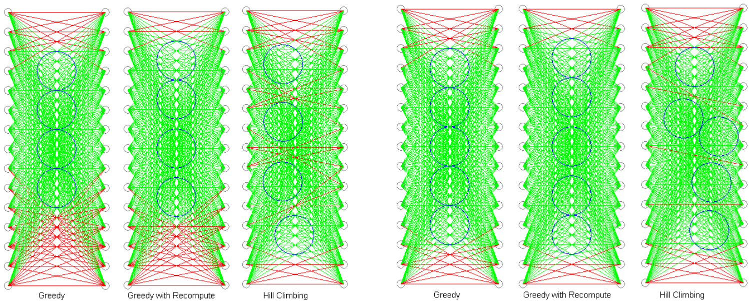
6 Buffon’s Needle Problem Reformulated

The Buffon’s Needle problem, first posed by the French naturalist Buffon in 1733 [1] is considered to be one of the best known problems in geometric probability [3, 4]. Imagine that a needle is dropped at random on the plane marked by equidistant parallel lines.



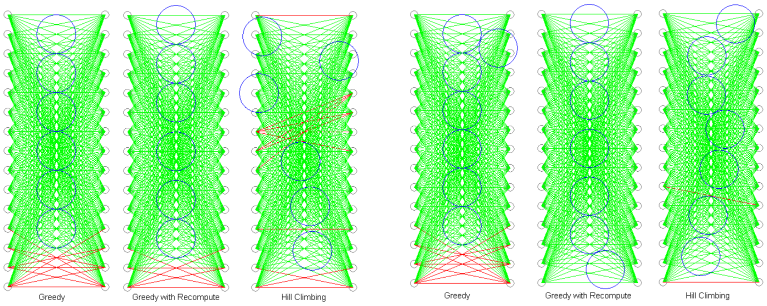
(a) 2 Tags (b) 3 Tags

Fig. 5. (a) 2-Tag Surface (b) 3-Tag Surface.



(a) 4 Tags (b) 5 Tags

Fig. 6. (a) 4-Tag Surface (b) 5-Tag Surface.



(a) 6 Tags (b) 7 Tags

Fig. 7. (a) 6-Tag Surface (b) 7-Tag Surface.

Let l be the length of the needle and let h be the distance between two consecutive lines. Buffon considered the case that $l < h$. For this case, the probability that the needle cuts at least one line can be shown to be $\frac{2l}{\pi h}$. The needle can be looked at as an imaginary line connecting two RFID readers placed on the robot. As the first approximation, the tag placement pattern can be a chessboard. The Buffon's Needle problem is reformulated as follows. A needle is dropped at random on a chessboard. What is the probability that the needle's two endpoints are in two cells of the same color? An interested reader is referred to [2] for the exact derivation of the formula. An optimal placement of RFID readers for a given robot is now chosen by maximizing the probability that at least two readers get valid readings.

Similar probability-theoretic formulations can be made over other placement patterns so long as each cell in the pattern is approximated with a known geometric shape: a circle, a triangle, an oval, etc. The quantification *at least two readers* in the optimization criterion should also be noted. There is nothing that prevents a robot to have more than two RFID readers. The practical considerations of cost and power consumption will act as the reality induced bounds on the probability-theoretic arguments.

7 Conclusions

In surface-embedded PRF exterocpetion, the cost of calibration is equivalent to the cost of deploying the PRF surface. Kepler's hexagonal packing pattern can be used to embed passive RFID transponders into horizontal surfaces. Proof-of-concept experiments show that such surfaces enable mobile robots to accomplish point-to-point navigation indoors and outdoors. Simulations show that dynamic greed compares favorably to brute force and hill climbing. The classic Buffon's Needle problem from computational geometry can be extended to create a feasible optimization pattern for packing PRF transponders into horizontal surfaces as long as the transponders' read areas can be respresented as circles with known radii. An optimal two-column pattern of arranging transponders on the edges of a surface occurs when the outer column is vertically shifted with respect to the inner column so that the horizontal line through each circle center in one column runs through the midpoint of two consecutive circle centers in the other column.

8 Acknowledgments

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References

1. G. Buffon. Editor's note concerning a lecture given 1733 by Mr. Le Clerc de Buffon to the Royal Academy of Sciences in Paris. *Histoire de l'Académie Royale des Sciences*, pages 43–45, 1733.

2. M. Jiang and V. Kulyukin. *Connect-the-Dots in a graph and Buffon's needle on a chessboard: two problems in assisted navigation*. Technical Report USU-CS-THEORY-2006-0309, Department of Computer Science, Utah State University, 2006.
3. D.A. Klain and G-C Rota, *Introduction to Geometric Probability*. Cambridge University Press, 1997.
4. A. M. Mathai, *An Introduction to Geometrical Probability: Distributed Aspects with Applications*. Gordon and Breach Science Publishers, 1999.
5. D. Fox, *Markov Localization: A Probabilistic Framework for Mobile Robot Localization and Navigation*. University of Bonn, Germany, 1998.
6. G. Kantor, and S. Singh, *Priliminary Results in Range-Only Localization and Mapping*. IEEE Conference on Robotics and Automation, Washington, D.C., May 2002.
7. T. Tsukiyama, *Navigation System for Mobile Robots using RFID tags*. IEEE Conference on Advanced Robotics, Coimbra, Portugal, June-July 2003.
8. J. Highthower, R. Want, and G. Borriello, *SpotOn: An Indoor 3D location sensing technology based on RF signal strength*. Technical Report, CSE 2000-02-02, University of Washington, Seattle, Washington, 2000.
9. O. Khatib, *Real-Time Obstacle Avoidance for Manipulators and Mobile Robots*. IEEE International Conference on Robotics and Automation, St. Louis, MO, 1985.
10. D. Hahnel, W. Burgard, D. Fox, K. Fishkin, and M. Philipose, *Mapping and localization with rfid technology*. Intel Research Institute. Tech. Rep. IRS-TR-03-014, Seattle, WA, 2003.
11. V. Kulyukin, C. Gharpure, J. Nicholson, *RoboCart: Toward Robot-Assisted Navigation of Grocery Stores by the Visually Impaired*. Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), July 2005.
12. A. Kutiyanawala, V. Kulyukin, E. LoPresti, J. Matthews, and R. Simpson, *A Rollator-Mounted Wayfinding System for the Elderly: A Smart World Perspective*. Proceedings of the 8th Conference on Computers and Accessibility (ASSETS 2006), Portland, Oregon.
13. V. Kulyukin, C. Gharpure, J. Nicholson, and G. Osborne, *Robot-Assisted Wayfinding for the Visually Impaired in Structured Indoor Environments*. Autonomous Robots, 21(1):29-41, 2006.
14. A. Burrell, *Robot lends a seeing eye for blind shoppers*. USA Today, Monday, Jul 11, 2005,
15. S. Willis and S. Helal, *A Passive RFID Information Grid for Location and Proximity Sensing for the Blind User*. University of Florida Technical Report number TR04-009, 2004.
16. R. J. Orr and G. D. Abowd, *The Smart Floor: A Mechanism for Natural User Identification and Tracking*. Georgia Institute of Technology, Technical Report number GIT-GVU-00-02, 2000.
17. H. Kautz, L. Arnstein, G. Borriello, O. Etzioni, D. Fox, *An Overview of the Assisted Cognition Project*. Proceedings of the 2002 AAAI Workshop on Automation as Caregiver: The Role of Intelligent Technology in Elder Care, 2002.
18. *The AMS Project* Autonomous Movement Support Project. http://www.ubin.jp/press/pdf/TEP040915-milt01e.pdf.
19. B. Casselman, *Packing Pennies in the Plane: An Illustrated Proof of Kepler's Conjecture in 2D*. http://www.math.sunysb.edu/ tony/whatsnew/column/pennies-1200/cass1.html.
20. C. Gharpure, V. Kulyukin, M. Jiang, and A. Kutiyanawala, *Passive Radio Frequency Exteroception in Robot Assisted Shopping for the Blind*. Proceedings of the 3rd International Conference on Ubiquitous Intelligence and Computing (UIC 2006), Wuhan, China, September 2006.